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NUMERICAL STUDIES OF SLOW VISCOUS ROTATING FLOW PAST A SPHERE. II

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SUMMARY

The Navier–Stokes equations, which are the governing equations for a steady, viscous, incompressible fluid rotating about the z-axis with angular velocity ω, are linearized using the Oseen approximation. Two parameters, namely the Reynolds number $Re = Ua/\nu$ and $Re_\omega = 2\omega a^2/\nu$ (the Reynolds number w.r.t. rotation), enter the linearized equations. These equations are solved by the Peaceman–Rachford ADI method and the resulting algebraic equations are solved by the SOR method. Streamlines are plotted and compared with the Oseen solution for the non-rotating case. The magnitude of the vorticity vector with increasing θ is also plotted.

KEY WORDS Peaceman–Rachford ADI method SOR method Oseen approximation

1. FORMULATION OF THE PROBLEM

The full Navier–Stokes[1] equations are linearized by taking $\psi = \psi_0 + \psi_1$ and $\Omega = \Omega_0 + \Omega_1$ and neglecting squares and products of ψ_1 and Ω_1 and their first-order partial derivatives. ψ_0 and Ω_0 are the streamfunction and rotational velocity (undisturbed) respectively. For an axisymmetric, steady, viscous fluid rotating about the z-axis, the linearized equations are

$$\left(D^2 - Re\frac{\partial}{\partial z}\right)D^2\psi_1 = -Re_\omega\left(\cos\theta\frac{\partial\Omega_1}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\Omega_1}{\partial\theta}\right), \tag{1}$$

$$\left(D^2 - Re\frac{\partial}{\partial z}\right)\Omega_1 = -Re_\omega\left(\cos\theta\frac{\partial\psi_1}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\Omega_1}{\partial\theta}\right), \tag{2}$$

where ψ_1 is the disturbed streamfunction, $\Omega_1 = r\sin\theta\ V_\phi$ is the disturbed rotational velocity, $\psi_0 = \frac{1}{2}r^2\sin^2\theta$ and $\Omega_0 = (C/2)r^2\sin^2\theta$, $C = 2a\omega/U$.

Equations (1) and (2) can be written as three coupled equations:

$$D^2\psi_1 = -r(\sin\theta)\zeta = -\zeta_1, \tag{3}$$

$$\left(D^2 - Re\frac{\partial}{\partial z}\right)\zeta_1 = -Re_\omega\left(\cos\theta\frac{\partial\Omega_1}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\Omega_1}{\partial\theta}\right), \tag{4}$$

$$\left(D^2 - Re\frac{\partial}{\partial z}\right)\Omega_1 = -Re_\omega\left(\cos\theta\frac{\partial\psi_1}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\psi_1}{\partial\theta}\right), \tag{5}$$

where ζ is the perturbed vorticity.

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Equations (3)–(5) are to be solved with the following boundary conditions:

$$\left.\begin{array}{l}\psi_1=\partial\psi_1/\partial r=0\\ \Omega_1=-(C/2)r^2\sin^2\theta\end{array}\right\}\quad \text{on}\quad r=1, \tag{6}$$

$$\left.\begin{array}{l}\psi_1\rightarrow 0\\ \Omega\rightarrow 0\end{array}\right\}\quad \text{as}\quad r\rightarrow\infty, \tag{7}$$

$$\psi_1=0\quad \text{for}\quad \theta=0,\quad 180^\circ \text{ axis of symmetry,} \tag{8}$$

$$\zeta=0\quad \text{for}\quad \theta=0,\quad 180^\circ \text{ axis of symmetry,} \tag{9}$$

$$\zeta\rightarrow 0\quad \text{as}\quad r\rightarrow\infty. \tag{10}$$

The conditions for ζ at the surface of the sphere have to be determined from the condition of zero velocity at the surface. i.e. $\partial\psi/\partial r=0$.

2. FINITE DIFFERENCE EQUATIONS

The finite difference method was given in Reference 1 and is not repeated here. Finite difference equations of $o(h^2)$ and $o(k^2)$ are written at each point (r_i, θ_j) for equations (3)–(5) and boundary conditions (6)–(10). We take $r=e^z$. With this substitution, $z=0$ corresponds to the sphere of radius $r=1$. The condition of infinity is taken as the container pipe of radius $r=e^2$. The boundary conditions (6)–(10) are written in finite difference form as

$$\left.\begin{array}{l}\psi_{1,0,j}=0\\ \Omega_{1,0,j}=-(C/2)\sin^2\theta_j\end{array}\right\}\quad \text{on}\quad z=0, \tag{11}$$

$$\left.\begin{array}{l}\psi_{1,N,j}=0\\ \Omega_{1,N,j}=0\\ \zeta_{1,N,j}=0\end{array}\right\}\quad \text{on}\quad z=2, \tag{12}$$

where we have taken $z_N=2$,

$$\zeta_{1,0,j}=-\frac{8\psi_{1,1,j}-\psi_{1,2,j}}{2h^2\sin\theta_j}. \tag{13}$$

Condition (13) is the condition of ζ_1 on the body. The finite difference equations of equations (3)–(5) with conditions (11)–(13) are solved using the Peaceman–Rachford ADI method given in Reference 1. In this method, in order to ensure diagonal dominance, the acceleration parameter ρ was chosen as 25. Twenty iterations were required at each stage to ensure convergence in the solution of equations (3)–(5) with boundary conditions (11)–(13).

Equation (5) had to be iterated 20 times before two successive iterated values of Ω_1 were coincident. These Ω_1 values were then used in equation (1.4), which was iterated 20 times to obtain ζ_1. Finally, these ζ_1 values were used in equation (3), which was iterated 20 times to obtain ψ_1. This process had to be repeated seven times before convergence was obtained. The starting values for ψ_1, Ω_1 and ζ_1 were taken as zero. A computer program for the Peaceman–Rachford ADI method where the resulting algebraic equations are solved by the SOR method has been developed on an IBM 370/155.

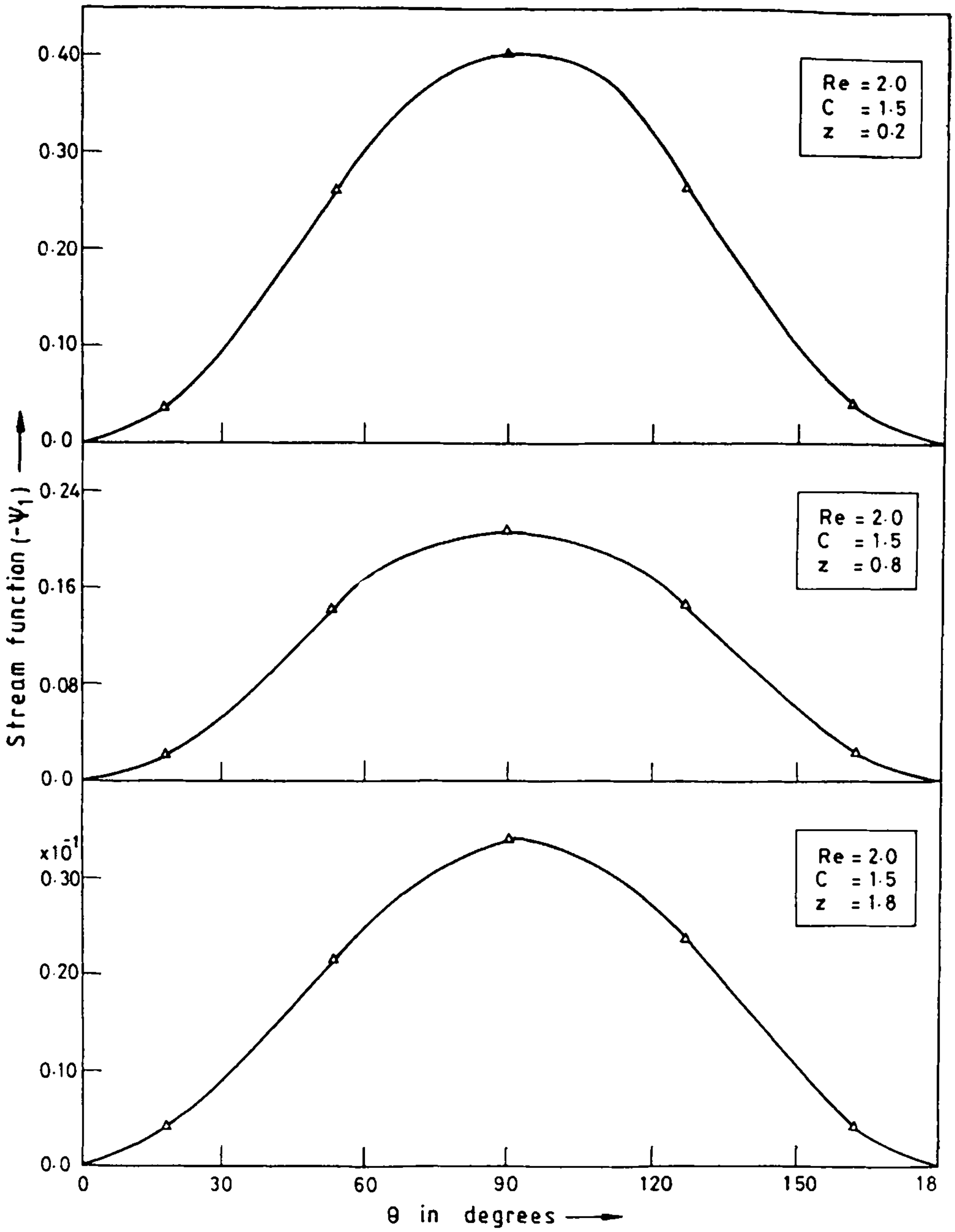
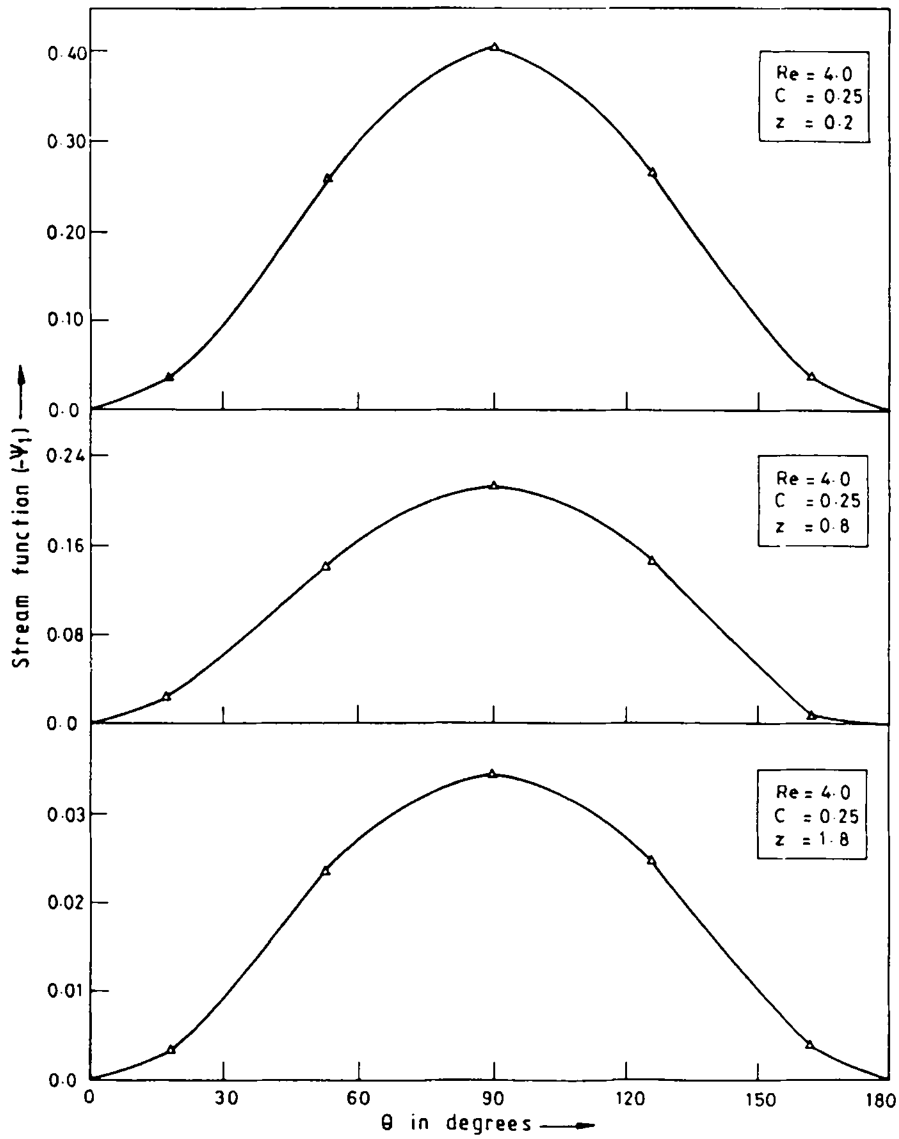


Figure 1

3. DISCUSSION OF RESULTS

In Figures 1–3 the streamfunction $(-\psi_1)$ is plotted against the angle θ for the rotating case. Figures 4–6 give the streamfunction $(-\psi_1)$ for the non-rotating case. The vorticity components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z = 0·2
Re = 4·0
C = 0·25
z = 0·8
Re = 4·0
C = 0·25
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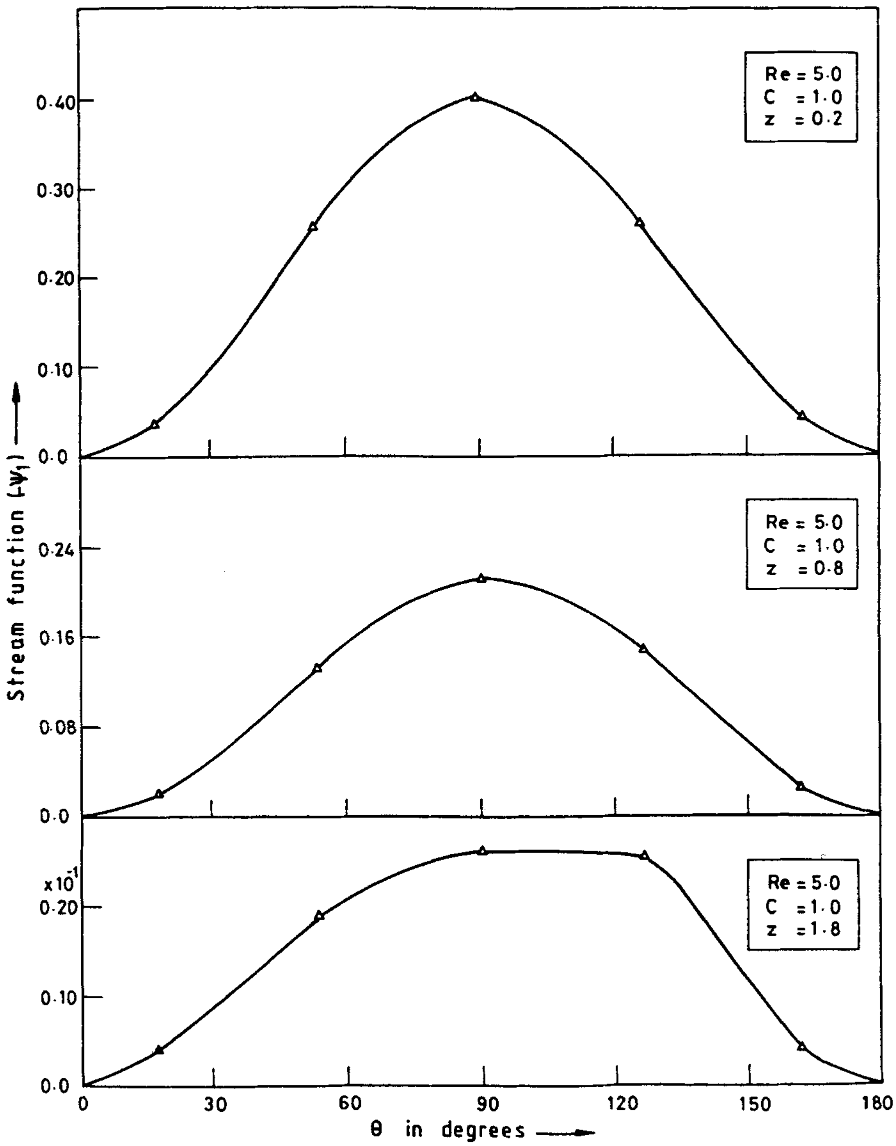
0·40
0·30
0·20
0·10
0·0
0·24
0·16
0·08
0·0
0·03
0·02
0·01
0·0
Stream function (-Ψ1)
0
30
60
90
120
150
180
θ in degrees

Figure 2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C = 1.0
z = 0.2
Re = 5.0
C = 1.0
z = 0.8
Re = 5.0
C = 1.0
z = 1.8
0.40
0.30
0.20
0.10
0.0
0.24
0.16
0.08
0.0
x10^-1
0.20
0.10
0.0
Stream function (-ψ1)
0
30
60
90
120
150
180
θ in degrees

Figure 3

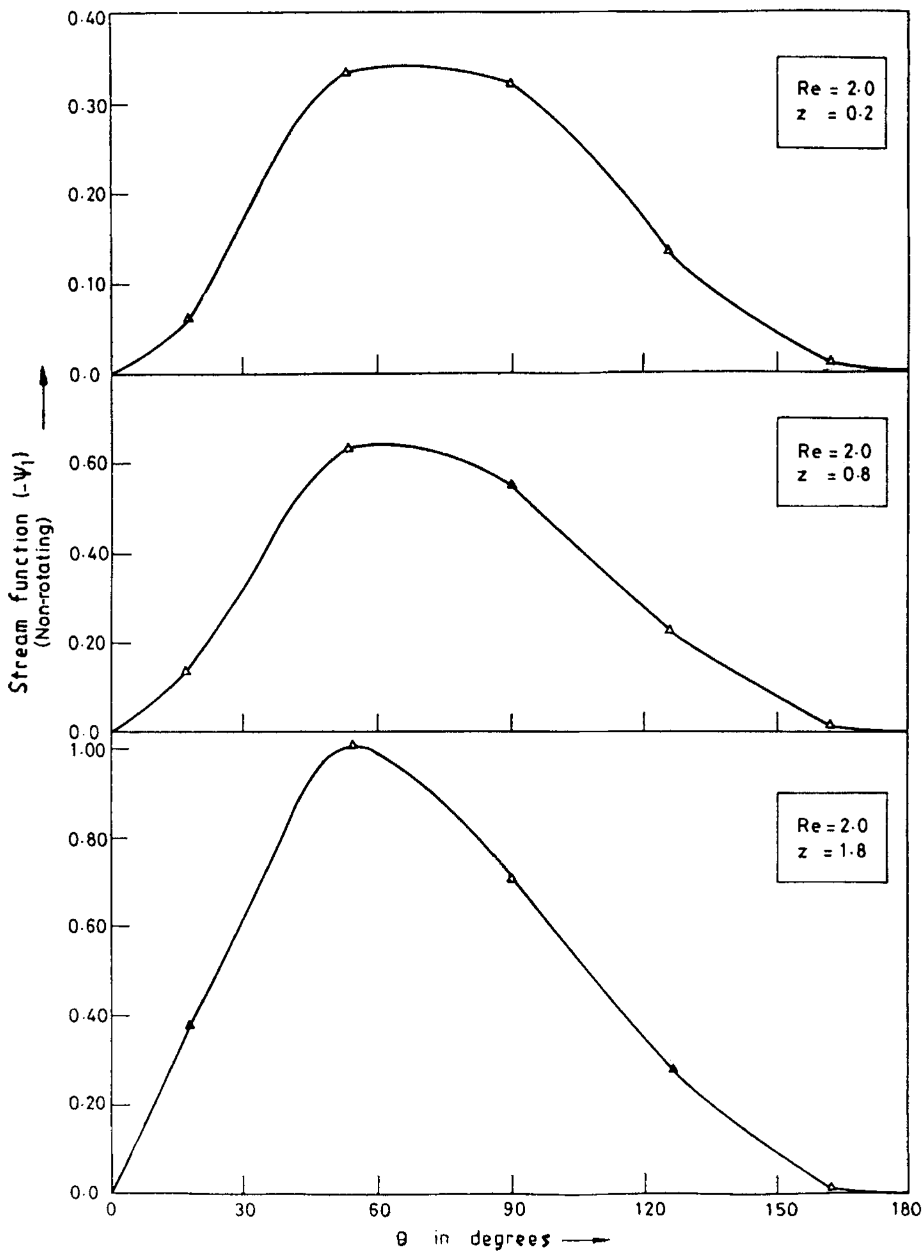

Re = 2.0
z = 0.2
Re = 2.0
z = 0.8
Re = 2.0
z = 1.8
Stream function (-ψ1)
(Non-rotating)
θ in degrees
0.40
0.30
0.20
0.10
0.0
0.60
0.40
0.20
0.0
1.00
0.80
0.60
0.40
0.20
0.0
0
30
60
90
120
150
180

Figure 4

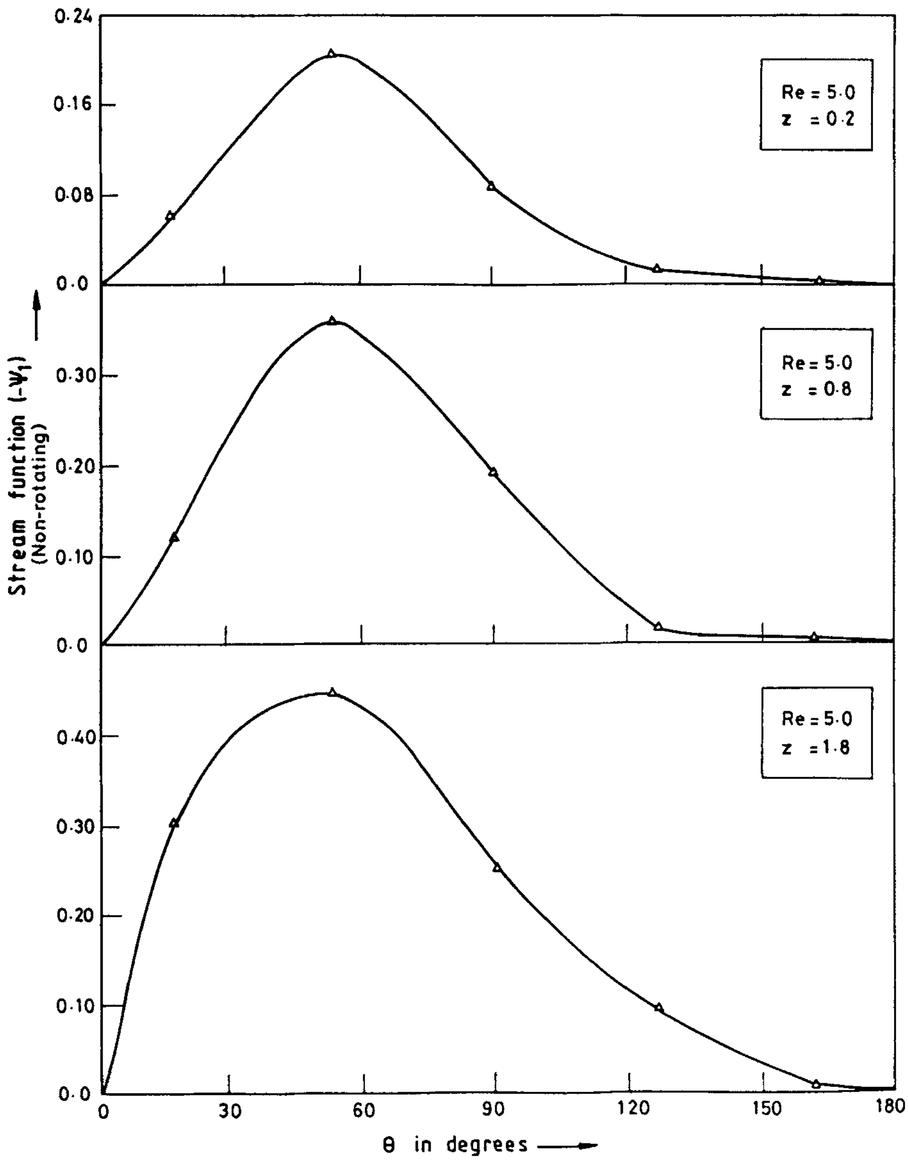
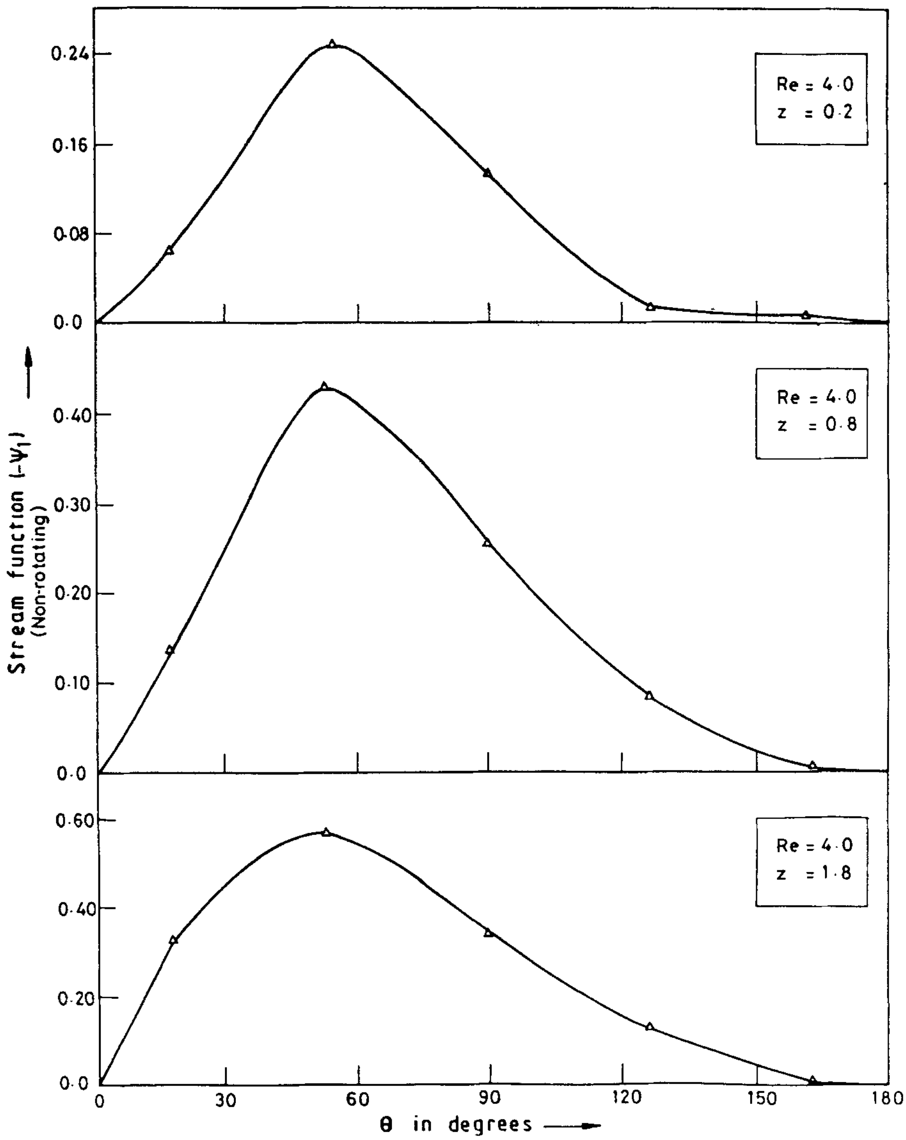

0.24
0.16
0.08
0.0
Re = 5.0
z = 0.2
0.30
0.20
0.10
0.0
Re = 5.0
z = 0.8
0.40
0.30
0.20
0.10
0.0
Re = 5.0
z = 1.8
Stream function (-Ψ1)
(Non-rotating)
0
30
60
90
120
150
180
θ in degrees

Figure 5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z = 0·2
Re = 4·0
z = 0·8
Re = 4·0
z = 1·8
0·24
0·16
0·08
0·0
0·40
0·30
0·20
0·10
0·0
0·60
0·40
0·20
0·0
0
30
60
90
120
150
180
θ in degrees
Stream function (−ψ1) (Non-rotating)

Figure 6

are given by

$$\xi = \frac{1}{r^2 \sin\theta} \frac{\partial \Omega_1}{\partial \theta}, \qquad \eta = \frac{1}{r \sin\theta} \frac{\partial \Omega_1}{\partial r}, \qquad \zeta = -\frac{1}{r \sin\theta} D^2 \psi_1.$$

Hence the magnitude of the vorticity vector is equal to $(\xi^2 + \eta^2 + \zeta^2)^{1/2}$.

In Figures 7–9, for the same values of Re and C, the magnitude of the vorticity vector for the rotating case is plotted against θ. In Figures 10 and 11 the streamlines are given for the rotating

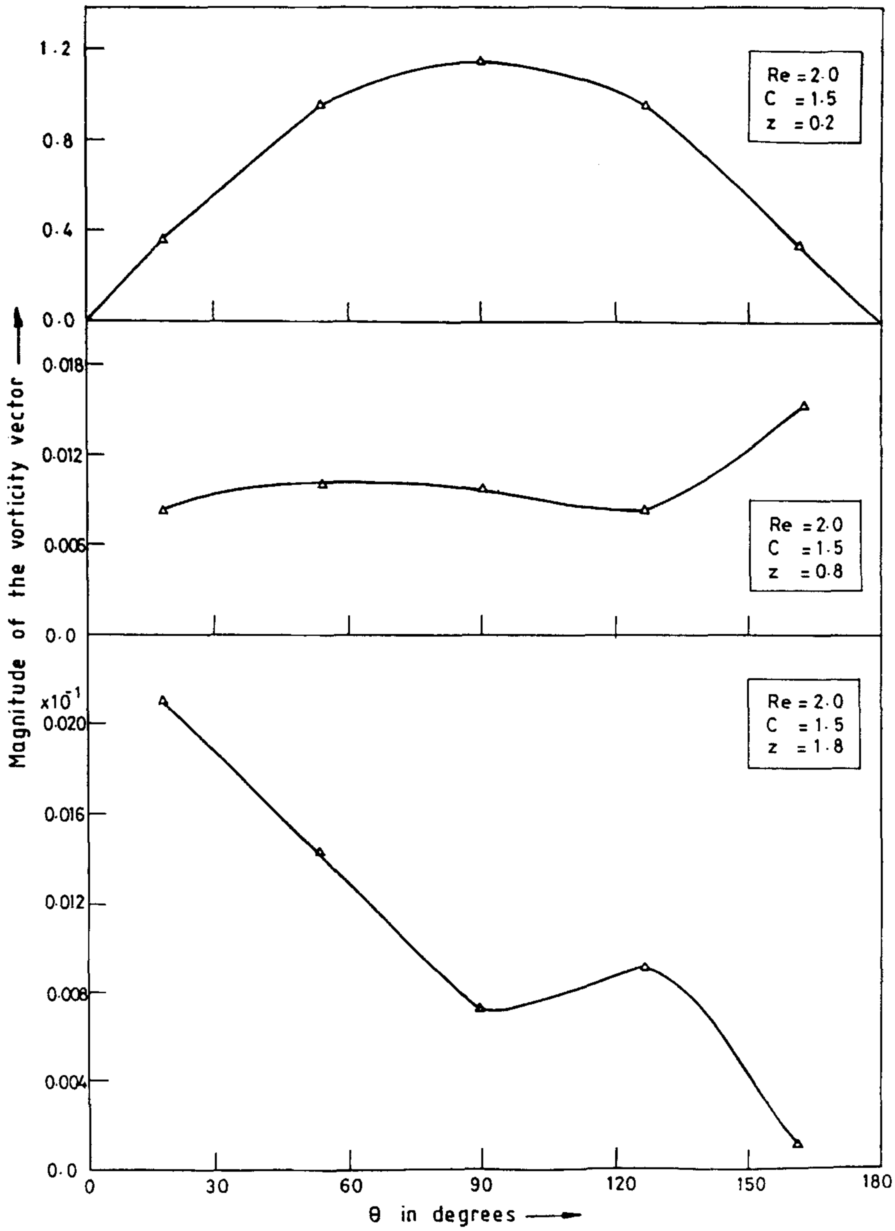
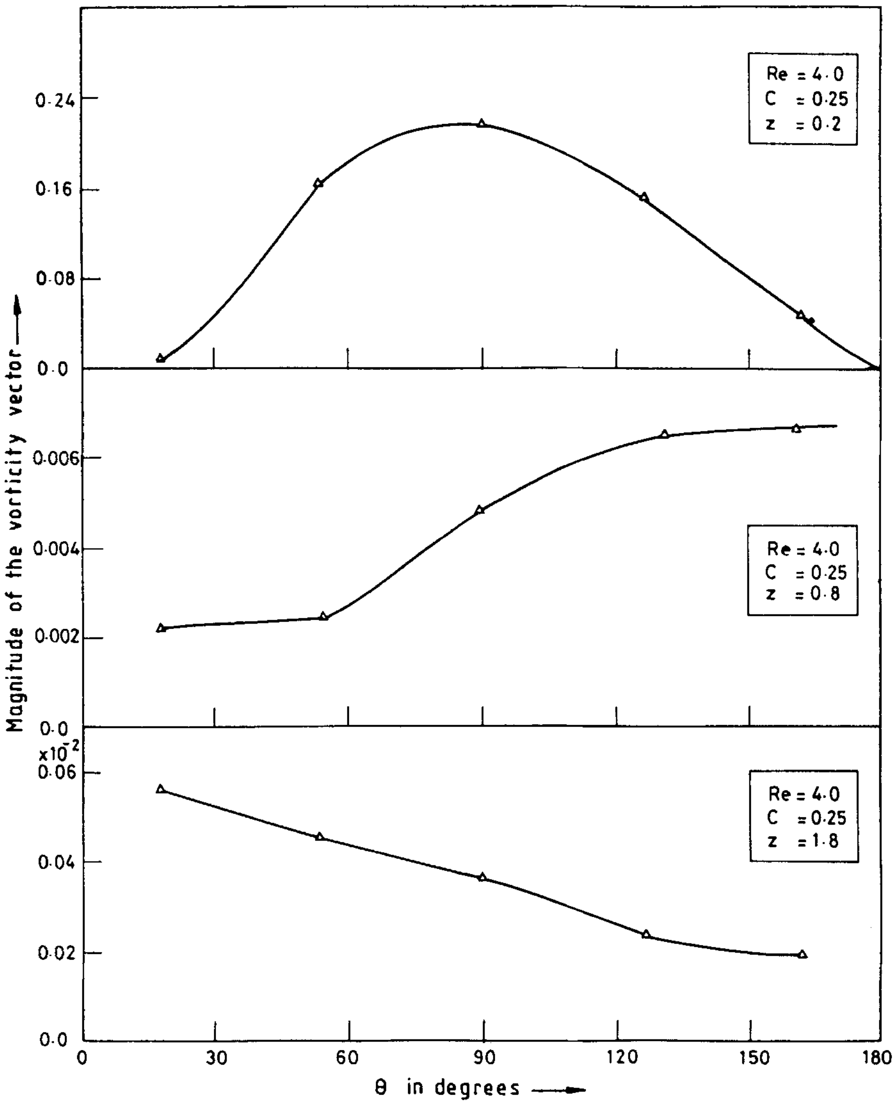


Figure 7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C = 0·25
z = 0·2
Re = 4·0
C = 0·25
z = 0·8
Re = 4·0
C = 0·25
z = 1·8
0·24
0·16
0·08
0·0
0·006
0·004
0·002
0·0
x10⁻²
0·06
0·04
0·02
0·0
0
30
60
90
120
150
180
Magnitude of the vorticity vector
θ in degrees

Figure 8

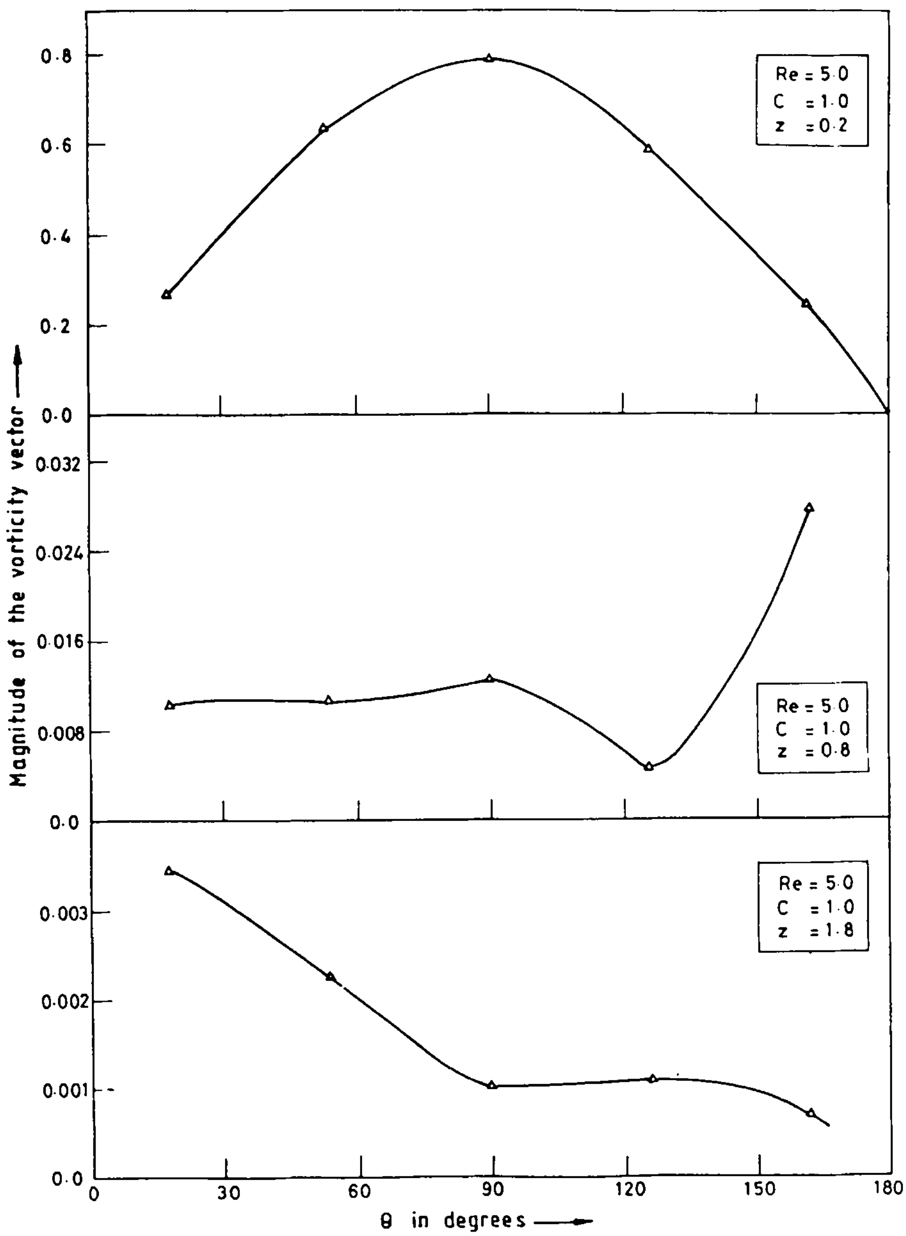
Re = 5.0
C = 1.0
z = 0.2
Re = 5.0
C = 1.0
z = 0.8
Re = 5.0
C = 1.0
z = 1.8
0.8
0.6
0.4
0.2
0.0
0.032
0.024
0.016
0.008
0.0
0.003
0.002
0.001
0.0
0
30
60
90
120
150
180
θ in degrees
Magnitude of the vorticity vector

Figure 9

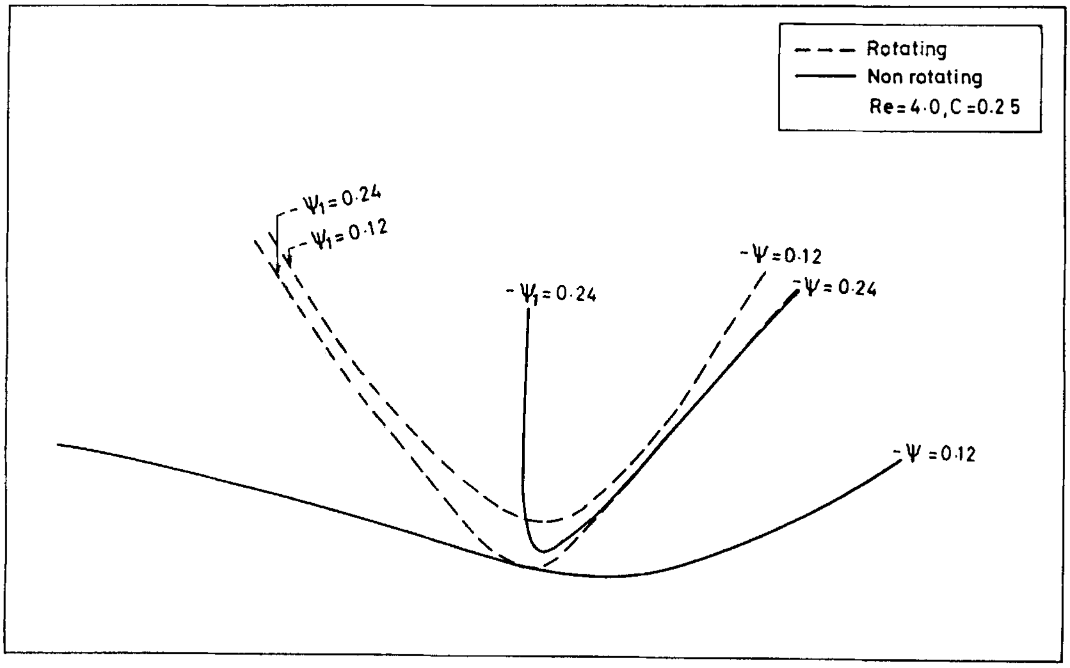

Figure 10

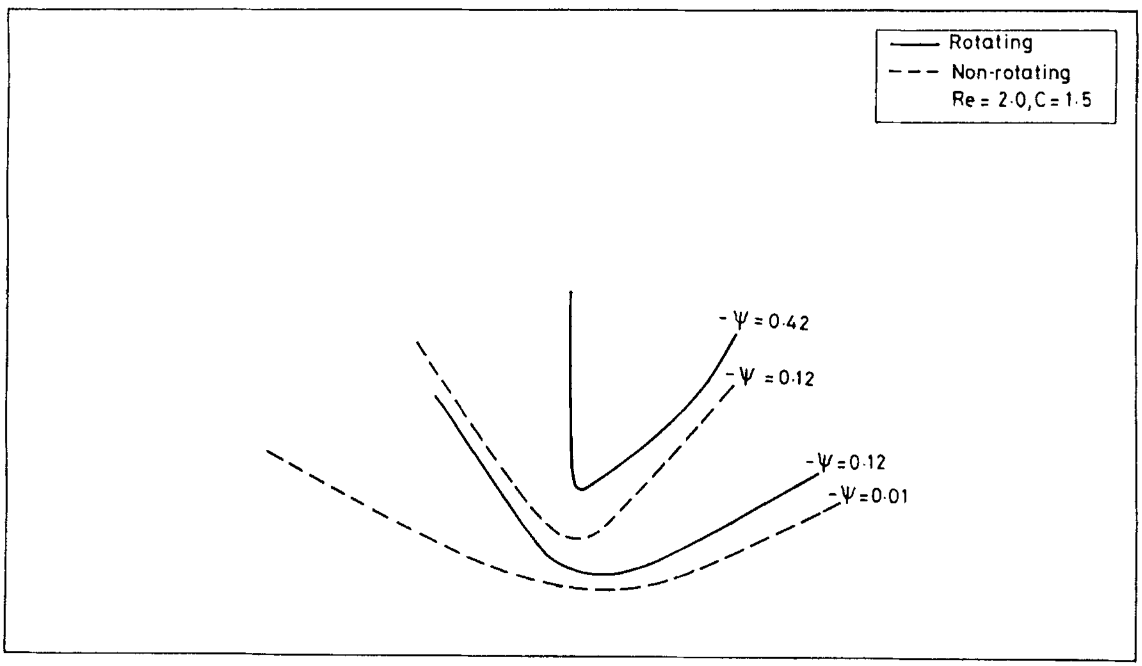

Figure 11

and non-rotating cases respectively. It is seen that the effect of rotation is to decrease the values of the streamfunction.

REFERENCES

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2. W. E. Langlois, *Slow Viscous Flow*, The Macmillan Company, New York, 1964, pp. 144–145.